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$\mathcal{N}=2$ super-Born-Infeld from partially broken $\mathcal{N}=3$ supersymmetry in $d=4$

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Abstract: We employ the non-linear realization techniques to relate the $\mathcal{N}=1$ chiral, and the $\mathcal{N}=2$ vector multiplets to the Goldstone spin 1/2 superfield arising from partial supersymmetry breaking of $\mathcal{N}=2$ and $\mathcal{N}=3$ respectively. In both cases, we obtain a family of non-linear transformation laws realizing an extra supersymmetry. In the $\mathcal{N}=2$ case, we find an invariant action which is the low energy limit of the supersymmetric Born-Infeld theory expected to describe a D3-brane in six dimensions.

Keywords: D-branes, Supersymmetry Breaking, Extended Supersymmetry, Supersymmetric Effective Theories.

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Contents

1. Introduction

During the second superstring revolution, we learned that the Dp-branes are the natural Ramond-Ramond charged objects [1]. This fact has revealed the importance of the Born-Infeld action and its possible supersymmetric extensions in diverse dimensions within the framework of string theory. Besides, the appearance of Yang-Mills in the effective Dp-brane stacks theories also set as a goal the consistent realization of Born-Infeld dynamics in the non-abelian case. In this sense, the last years bore witness to the second Born-Infeld revolution. It has been conjectured [2, 3] that supersymmetric Born-Infeld theories emerge naturally form partial supersymmetry breaking (PSSB), as it has been proposed and successfully tested in several works for particular cases spanning different dimensions and number of supersymmetries [2]–[11]. The usual procedure consists in imposing irreducibility conditions on the Goldstone fields arising from the PSSB to select the particular representation of the unbroken residual symmetry in which they lie. Former studies assigned this Goldstone superfield to the $\mathcal{N} = 1$ chiral [12] and Maxwell [6] multiplets. In the $\mathcal{N} = 2$ case, the vector supermultiplet has also been interpreted as the Goldstone multiplet coming from two broken supersymmetries [9, 11]. Particularly in [3], the authors make use of the Goldstone bosonic $\mathcal{N} = 2$ and $\mathcal{N} = 4$ superfields associated with the breaking of $\mathcal{N} = 4$ and $\mathcal{N} = 8$ central charge generators respectively. The PSSB mechanism has been also studied within the framework of M-theory and $d = 11$ supergravity using the superembedding techniques [13].

The common mathematical framework used to describe PSSB is the non-linear realization method originally developed by Callan, Coleman, Wess, and Zumino [14, 15] for semi-simple internal symmetries, and extended to supersymmetry by Akulov and Volkov [16]. In this work we use non-linear realizations, in a complete off-shell fashion, to derive a general family of non-linear transformation laws for the chiral Goldstone superfield coming from PSSB of $\mathcal{N} = 2$ supersymmetry without central charges down to $\mathcal{N} = 1$. As special cases, we find the action obtained by Bagger and Galperin [12], and the one obtained by Roček and Tseytlin [20] up to fourth order in the chiral superfield which is the translational invariant action of the $\mathcal{N} = 1$ 3-brane proposed by Hughes and Polchinski in [21].

Later on we deal with the PSSB in the $\mathcal{N} = 3 \longrightarrow \mathcal{N} = 2$ case, where we obtain a $\mathcal{N} = 2$ action for the vector multiplet which is invariant under the extra hidden supersymmetry. We impose the proper covariant irreducibility conditions over the spin 1/2 Goldstone superfield to relate it to the $\mathcal{N} = 2$ chiral superfield containing the vector multiplet. In this case we obtain also a family of transformation laws as well as their invariant actions. In a particular case, the lagrangian has exactly the terms that come from the Born-Infeld low energy limit up to $\mathrm{O}(F^6)$. This action has been interpreted as the world-volume dynamics of a D3-brane propagating in six dimensions [17]. The properties required for such model have been studied in [10]. It is important to remark that our derivation is in both cases off-shell.

2. Non-linear realizations in superspace

In this section, we briefly review the standard non-linear realizations formalism for the $\mathcal{N} = 2 \longrightarrow \mathcal{N} = 1$ case. Let G be the super-Poincaré $\mathcal{N} = 2$ group which we would like to breakdown and $H \subset G$ the unbroken invariant subgroup. Consider the splitting of the generators of G into three classes: The generators of superspace translations $\Gamma_{\underline{A}} = (P_m, Q_\alpha, \bar{Q}^{\dot{\alpha}})$, where Q_α are the residual supersymmetry charges, the broken symmetry generators $\Gamma_r = (S_\alpha, \bar{S}^{\dot{\alpha}})$, and the preserved symmetry generators Γ_i belonging to the Lie algebra of H which corresponds in our case to the Lorenz subgroup $SO(3,1)$.

Starting from the $\mathcal{N} = 2$ algebra without central charges

$$\begin{aligned}
\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} &= 2\sigma^m_{\alpha\dot{\beta}} P_m\,, \\
\{S_\alpha, \bar{S}_{\dot{\beta}}\} &= 2\sigma^m_{\alpha\dot{\beta}} P_m\,, \\
\{Q_\alpha, \bar{S}_{\dot{\beta}}\} &= 0\,, \\
\{Q_\alpha, S_\beta\} &= 0\,,
\end{aligned} \tag{2.1}$$

the parameterization of the coset space G/H is taken to be

$$\Omega = \exp\left(\mathrm{i} X^{\underline{A}} \Gamma_{\underline{A}}\right) \exp(\mathrm{i}\psi^\alpha S_\alpha + \mathrm{i}\bar{\psi}_{\dot{\alpha}} \bar{S}^{\dot{\alpha}})\,. \tag{2.2}$$

Here $\psi^\alpha = \psi^\alpha(x, \theta, \bar{\theta})$ is the Goldstone *superfield* related to the breaking of the S_α generator.

The action of $g = \exp(\mathrm{i}\eta S + \mathrm{i}\bar{\eta}\bar{S})$, where η^α is a constant spinor parameter, over a general element of the coset space can be understood as a transformation law for the Goldstone fields and the coordinates

$$\begin{aligned}(x'^m, \theta', \bar{\theta}') &= (x^m + \mathrm{i}(\eta\sigma^m\bar{\psi} - \psi\sigma^m\bar{\eta}), \theta, \bar{\theta}) \,, \\ \psi' &= \psi + \eta \,, \\ \bar{\psi}' &= \bar{\psi} + \bar{\eta} \,, \end{aligned} \tag{2.3}$$

from which we can derive the non-linear transformation law for the Goldstone field

$$\delta\psi_\alpha = \eta_\alpha - \mathrm{i}(\eta\sigma^m\bar{\psi} - \psi\sigma^m\bar{\eta})\partial_m\psi \,. \tag{2.4}$$

We are now able to deduce the covariant $\mathcal{N} = 1$ 1-forms

$$\begin{aligned}\omega^m(P) &= dx^m + \mathrm{i}(d\theta\sigma^m\bar{\theta} - \theta\sigma^m d\bar{\theta}) + \mathrm{i}(d\psi\sigma^m\bar{\psi} - \psi\sigma^m d\bar{\psi}) \,, \\ \omega^\alpha(Q) &= d\theta^\alpha \,, \\ \bar{\omega}_{\dot{\alpha}}(\bar{Q}) &= d\bar{\theta}_{\dot{\alpha}} \,, \\ \omega^\alpha(S) &= d\psi^\alpha \,, \\ \bar{\omega}_{\dot{\alpha}}(\bar{S}) &= d\bar{\psi}_{\dot{\alpha}} \,, \end{aligned} \tag{2.5}$$

that assemble into the invariant Maurer-Cartan 1-form

$$\Omega^{-1}d\Omega = \mathrm{i}[\omega^m(P)P_m + \omega^\alpha(Q)Q_\alpha + \bar{\omega}_{\dot{\alpha}}(\bar{Q})\bar{Q}^{\dot{\alpha}} + \omega^\alpha(S)S_\alpha + \bar{\omega}_{\dot{\alpha}}(\bar{S})\bar{S}^{\dot{\alpha}}] \,. \tag{2.6}$$

The expansion of the first three 1-forms in equation (2.5) $\omega^{\underline{A}} = dX^{\underline{M}}E_{\underline{M}}^{\underline{A}}$ in terms of $dX^{\underline{A}} = (dx^m, d\theta^\alpha, d\bar{\theta}_{\dot{\alpha}})$, lead us to the supervierbein

$$\begin{aligned}E_n^m &= \delta_n^m + \mathrm{i}(\partial_n\psi\sigma^m\bar{\psi} - \psi\sigma^m\partial_n\bar{\psi}) \,, \\ E_\beta^m &= \mathrm{i}(\sigma^m\bar{\theta})_\beta + \mathrm{i}(\partial_\beta\psi\sigma^m\bar{\psi} + \psi\sigma^m\partial_\beta\bar{\psi}) \,, \\ E^{\dot{\beta}m} &= \mathrm{i}(\bar{\sigma}^m\theta)^{\dot{\beta}} + \mathrm{i}(\bar{\partial}^{\dot{\beta}}\psi\sigma^m\bar{\psi} + \psi\sigma^m\bar{\partial}^{\dot{\beta}}\bar{\psi}) \,, \\ E_\beta^\alpha &= \delta_\beta^\alpha \,, \\ E_{\dot{\alpha}}^{\dot{\beta}} &= \delta_{\dot{\alpha}}^{\dot{\beta}} \,. \end{aligned} \tag{2.7}$$

In the same fashion one must expand the remaining 1-forms $\omega^\alpha = \omega^{\underline{M}}\mathcal{D}_{\underline{M}}\psi^\alpha$, to find the covariant derivatives of the Goldstone superfield $\mathcal{D}_{\underline{M}}\psi^\alpha = E^{-1}{}_{\underline{M}}{}^{\underline{N}}\partial_{\underline{N}}\psi^\alpha$. This calculation involves the inversion of the supervierbein matrix

$$E^{-1}{}_{\underline{M}}{}^{\underline{N}} = \begin{pmatrix} \omega^{-1}{}_m{}^n & -E_\alpha^p\omega^{-1}{}_p{}^n & -E^{\dot{\alpha}p}\omega^{-1}{}_p{}^n \\ 0 & \delta_\alpha^\beta & 0 \\ 0 & 0 & \delta_{\dot{\beta}}^{\dot{\alpha}} \end{pmatrix} , \tag{2.8}$$

which is given in terms of the direct supervierbein and the inverse of $\omega_n^m = \delta_n^m + \mathrm{i}(\partial_n\psi\sigma^m\bar{\psi} - \psi\sigma^m\partial_n\bar{\psi})$. With these elements we can express the covariant derivatives in a seemingly explicit form:

$$\begin{aligned}
\mathcal{D}_m &= \omega_m^{-1\,n}\partial_n\,,\\
\mathcal{D}_\alpha &= D_\alpha - \mathrm{i}(D_\alpha\psi\sigma^m\bar{\psi} + \psi\sigma^m D_\alpha\bar{\psi})\mathcal{D}_m\,,\\
\bar{\mathcal{D}}_{\dot\alpha} &= \bar{D}_{\dot\alpha} - \mathrm{i}(\bar{D}_{\dot\alpha}\psi\sigma^m\bar{\psi} + \psi\sigma^m \bar{D}_{\dot\alpha}\bar{\psi})\mathcal{D}_m\,,
\end{aligned} \tag{2.9}$$

where D_α and $\bar{D}_{\dot\alpha}$ are the usual superspace derivatives. (See appendix A for conventions). By defining irreducibility constraints over chiral fields with the aid of covariant derivatives we will be able to treat the ghost states problem of PSSB requiring the Goldstone multiplet to be an $\mathcal{N}=1$ irreducible *and* $\mathcal{N}=2$ covariant representation.

3. The family of non-linear transformation laws and the $\mathcal{N}=1$ invariant actions

Generally, the chiral multiplet can be associated with the Goldstone spin-1/2 field imposing constraints on the components of a $\mathcal{N}=1$ SUSY irrep [18, 19]

$$\bar{D}\bar{D}\psi_\alpha\,,\qquad \bar{D}\bar{D}D_\beta\psi_\alpha\,,\qquad \bar{D}\bar{D}DD\psi_\alpha\,. \tag{3.1}$$

To specifically cancel out the irrelevant pieces of the superfield keeping only the chiral part one must choose [5]

$$\begin{aligned}
\bar{D}\bar{D}\psi_\alpha &= 0\,,\\
D_\alpha\psi_\beta + D_\beta\psi_\alpha &= 0\,,
\end{aligned} \tag{3.2}$$

as to select only the antisymmetric term $\bar{D}\bar{D}D_{[\beta}\psi_{\alpha]}$ leading to the chiral multiplet. This constraint is readily solved by taking $\psi_\alpha = \lambda D_\alpha\Phi$, where Φ is a chiral field which non-linearly realize a second supersymmetry if its chirality condition is expressed in terms of covariant derivatives. The parameter λ of geometrical dimension $[\lambda]=-2$ will be related to the scale of supersymmetry breaking. The curve chirality condition can be written as

$$\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0 \tag{3.3}$$

This solves the proper constraints (3.2) in the flat limit $\mathcal{D}\longrightarrow D$. The solution of this generalized constraint is defined up to a chiral field $\bar{D}^2\mathcal{X}$.

$$\Phi = \varphi - \mathrm{i}\lambda^2(D\varphi\sigma^m\bar{D}\bar{\varphi})\partial_m\varphi + 4\lambda^2\bar{\varphi}(\partial\varphi)^2 + \lambda^2\bar{D}^2\mathcal{X} + \mathrm{O}(\lambda^4)\,. \tag{3.4}$$

$\mathcal{X}$ has geometrical dimension $[\mathcal{X}]=4$ and will be constructed out of φ to preserve $\mathcal{N}=1$ supersymmetry. Non-linear transformation laws for the fields can then be determined up to a superfield Z_ξ containing both φ and the parameter of the transformation ξ.

$$\delta\Phi = \theta\xi - \mathrm{i}\lambda^2(\xi\sigma^m\bar{D}\bar{\varphi} - D\varphi\sigma^m\bar{\xi})\partial_m\varphi + \mathrm{O}(\lambda^4)\,, \tag{3.5a}$$

$$\delta\varphi = \theta\xi + 2\mathrm{i}\lambda^2(D\varphi\sigma^m\bar{\xi})\partial_m\varphi - 4\lambda^2(\partial\varphi)^2(\bar{\theta}\bar{\xi}) - \lambda^2\bar{D}^2\delta\mathcal{X} + \lambda^2D^2Z_\xi + \mathrm{O}(\lambda^4)\,. \tag{3.5b}$$

Considering the liberty of choice of $\mathcal{X}$ and Z_ξ we actually have a family of non-linear transformation laws, which together with the fields, have to fulfill the irreducibility constraints at least in the linear limit. In addition, the fields $\mathcal{X}$ and Z_ξ must also be constrained so that the algebra of the non-linear transformations correspond to that of an extra supersymmetry. This amounts to the following equations:

$$\bar{D}^2[\delta_\xi, \delta_\eta]\mathcal{X} = D^2\delta_{[\xi}Z_{\eta]}\,, \tag{3.6a}$$
$$\bar{D}_{\dot\alpha}D^2 Z_\xi = 0\,. \tag{3.6b}$$

These conditions set $Z_\xi = 0$ (see Appendix B) but allow $\mathcal{X}$ to be a very general field. Some admissible solutions are

$$\mathcal{X} = a_1\varphi(D\varphi)^2 + a_2\bar\varphi(D\varphi)^2 + a_3\varphi\bar\varphi\bar{D}^2\bar\varphi + a_4\bar\varphi(\bar{D}\bar\varphi)^2 + a_5\varphi^2 D^2\varphi + a_6\varphi\bar\varphi D^2\varphi + \\ + a_7\bar\varphi^2 D^2\varphi + a_8\bar\varphi^2\bar{D}^2\bar\varphi\,, \tag{3.7}$$

the superalgebra corresponds to

$$[\delta_\xi, \delta_\eta]\varphi = 2\mathrm{i}\lambda^2(\xi\sigma^m\bar\eta - \eta\sigma^m\bar\xi)\partial_m\varphi + \mathrm{O}(\lambda^4)\,. \tag{3.8}$$

The parameters a_i span the set of superfields defining a family of non-linear transformation laws. Now we are ready to build a $\mathcal{N} = 1$ action with a non-linearly realized extra supersymmetry. We start with the usual $\mathcal{N} = 1$ chiral action

$$S_1 = \int d^4x\, d^2\theta\, d^2\bar\theta\, \varphi\bar\varphi\,. \tag{3.9}$$

Its variation with respect to the transformation law (3.5b) is then

$$\delta S_1 = \int d^4x\, d^2\theta\, d^2\bar\theta\, \left[(\theta\xi)\bar\varphi + 2\mathrm{i}\lambda^2(D\varphi\sigma^m\bar\xi)\partial\varphi\bar\varphi - 4\lambda^2(\partial\varphi)^2(\bar\theta\bar\xi)\bar\varphi - \lambda^2\bar{D}^2\delta\mathcal{X}\bar\varphi\right] + \\ + \text{c.c.} + \mathrm{O}(\lambda^4)\,. \tag{3.10}$$

Due to the fact that $(\theta\xi)\bar\varphi$, $\bar{D}^2\mathcal{X}(\bar\theta\bar\xi)$ and their complex conjugates do not contribute to the integral above, we can express the variation of S_1 as the first order variation of terms of second order in λ

$$\delta S_1 = \lambda^2\delta\int d^4x\, d^2\theta\, d^2\bar\theta\left[\frac{1}{4}(D\varphi)^2(\bar{D}\bar\varphi)^2 - 2(\partial\varphi)^2\bar\varphi^2 - 2(\partial\bar\varphi)^2\varphi^2 + \bar{D}^2\mathcal{X}\bar\varphi + D^2\bar{\mathcal{X}}\varphi\right] + \\ + \mathrm{O}(\lambda^4)\,. \tag{3.11}$$

Then it is a simple task to propose a set of $\mathcal{N} = 1$ chiral gauge actions with an extra non-linear supersymmetry

$$\hat{S}_1 = \int d^4x\, d^2\theta\, d^2\bar\theta\Big[\varphi\bar\varphi + 2\lambda^2(\partial\varphi)^2\bar\varphi^2 + 2\lambda^2(\partial\bar\varphi)^2\varphi^2 - \\ - \frac{1}{4}\lambda^2(D\varphi)^2(\bar{D}\bar\varphi)^2 + \lambda^2\bar{D}^2\mathcal{X}\bar\varphi + \lambda^2 D^2\bar{\mathcal{X}}\varphi\Big] + \mathrm{O}(\lambda^4)\,. \tag{3.12}$$

Choosing $\mathcal{X} = 0$ we obtain the action of Bagger and Galperin [12], coming from the breaking of $\mathcal{N} = 2$ supersymmetry with central charges. Note that our conventions are different from the cited paper (see appendix A). We can further impose phase invariance of the action

$$a_1 = a_3 = a_4 = a_5 = a_7 = a_8 = 0\,. \tag{3.13}$$

The $N = 1$ chiral action (3.12) with this constraints will then be equivalent to

$$\hat{S}_1 = \int d^4x\, d^2\theta\, d^2\bar{\theta} \Big[\varphi\bar{\varphi} + 2(1 + 4a_2 + 8a_6)\lambda^2 \left[(\partial\varphi)^2\bar{\varphi}^2 + (\partial\bar{\varphi})^2\varphi^2\right] - \\ - \left(\frac{1}{4} + 2a_2 - 2a_6\right)\lambda^2(D\varphi)^2(\bar{D}\bar{\varphi})^2 + \frac{1}{4}a_6\lambda^2\bar{D}^2\bar{\varphi}^2D^2\varphi^2\Big] + \mathrm{O}(\lambda^4)\,.$$

Choosing $a_2 = -\frac{1}{4}$ and $a_6 = 0$ we find the action in [20] up to $\mathrm{O}(\varphi^4)$ which is the translational invariant action of the $\mathcal{N} = 1$ 3-brane proposed in [21]. It is important to comment that all these actions are equivalent through field redefinitions. Nevertheless, we have used the generalized family of transformation laws (3.5b) to obtain the actions in [12] and [20]. Our analysis suggests that the nature of the transformation laws (3.5b) is the origin of the equivalence between these dynamics.

4. Partial $\mathcal{N} = 3$ supersymmetry breaking

In this section, we construct a $\mathcal{N} = 2$ action for the vector multiplet which is invariant under a third non-linearly realized supersymmetry. This action contains non-linear terms in the spin one gauge fields which are interpreted as the low energy limit of the supersymmetric Born-Infeld theory. We repeat the procedure of section section 2 but enlarging the superspace by one Grassmann variable (X^m, θ_A) with $\theta_A = (\theta, \tilde{\theta})$. This notation will be understood for every Grassmann variable or operator i.e. $D^A_\alpha = (D_\alpha, \tilde{D}_\alpha)$. Indexes A, B are SU(2) and therefore raised and lowered with ϵ_{AB}. We will represent the volume element in the Grassmann variables by

$$\begin{aligned} d\Theta &= d^2\theta d^2\tilde{\theta}\,, & d\bar{\Theta} &= d^2\bar{\theta}d^2\bar{\tilde{\theta}}\,, \\ d^4\theta &= d^2\theta d^2\bar{\theta}\,, & d^4\tilde{\theta} &= d^2\tilde{\theta}d^2\bar{\tilde{\theta}}\,. \end{aligned} \tag{4.1}$$

In this case the total group G is the $\mathcal{N} = 3$ supersymmetry group and H is still $SO(3,1)$ The $\mathcal{N} = 3$ algebra without central charges is

$$\begin{aligned} \{Q^A_\alpha, \bar{Q}_{\dot{\beta}B}\} &= 2\delta^A_B\sigma^m_{\alpha\dot{\beta}}P_m\,, \\ \{S_\alpha, \bar{S}_{\dot{\beta}}\} &= 2\sigma^m_{\alpha\dot{\beta}}P_m\,, \\ \{Q^A_\alpha, \bar{S}_{\dot{\beta}}\} &= 0\,, \\ \{Q^A_\alpha, S_\beta\} &= 0\,, \end{aligned} \tag{4.2}$$

the parameterization of the coset space G/H has the same form as the former case

$$\Omega = \exp(\mathrm{i}X^r\Gamma_r)\exp(\mathrm{i}\psi^\alpha S_\alpha + \mathrm{i}\bar{\psi}_{\dot{\alpha}}\bar{S}^{\dot{\alpha}})\,. \tag{4.3}$$

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Now $\psi^\alpha = \psi^\alpha(x, \theta_A, \bar\theta^A)$ is the Goldstone *superfield* related to the breaking of the S_α generator. Though the transformation law for the Goldstone superfield is analogous to (2.4), the supervierbein is in this case

$$
\begin{aligned}
E_n^m &= \delta_n^m + \mathrm{i}(\partial_n\psi\sigma^m\bar\psi - \psi\sigma^m\partial_n\bar\psi)\,, \\
E_\beta^{mA} &= \mathrm{i}(\sigma^m\bar\theta^A)_\beta + \mathrm{i}(\partial_\beta^A\psi\sigma^m\bar\psi + \psi\sigma^m\partial_\beta^A\bar\psi)\,, \\
E_A^{\dot\beta m} &= \mathrm{i}(\bar\sigma^m\theta_A)^{\dot\beta} + \mathrm{i}(\bar\partial_A^{\dot\beta}\psi\sigma^m\bar\psi + \psi\sigma^m\bar\partial_A^{\dot\beta}\bar\psi)\,, \\
E_\beta^\alpha &= \delta_\beta^\alpha\,, \qquad \tilde E_\beta^\alpha = \delta_\beta^\alpha\,, \\
E_{\dot\alpha}^{\dot\beta} &= \delta_{\dot\alpha}^{\dot\beta}\,, \qquad \tilde E_{\dot\alpha}^{\dot\beta} = \delta_{\dot\alpha}^{\dot\beta}\,.
\end{aligned}
\tag{4.4}
$$

The covariant non-linear derivatives of the Goldstone superfield $\mathcal{D}_{\underline{M}}^A\psi^\alpha = (E_{\underline{M}}^{A\,\underline{N}})^{-1}\partial_{\underline{N}}\psi^\alpha$ are

$$
\begin{aligned}
\mathcal{D}_m &= \omega_m^{-1\,n}\partial_n\,, \\
\mathcal{D}_\alpha^A &= D_\alpha^A - \mathrm{i}(D_\alpha^A\psi\sigma^m\bar\psi + \psi\sigma^m D_\alpha^A\bar\psi)\mathcal{D}_m\,, \\
\bar{\mathcal{D}}_{\dot\alpha A} &= \bar D_{\dot\alpha A} - \mathrm{i}(\bar D_{\dot\alpha A}\psi\sigma^m\bar\psi + \psi\sigma^m\bar D_{\dot\alpha A}\bar\psi)\mathcal{D}_m\,.
\end{aligned}
\tag{4.5}
$$

Starting with the $\mathcal{N}=2$ superfield $\mathcal{W} = \mathcal{W}(x, \theta^A, \bar\theta^B)$, it is possible to build an irreducible representation of the $\mathcal{N}=2$ supersymmetry in terms of $\mathcal{N}=1$ superfields by imposing the following chirality and reality conditions [22],

$$
\bar D_{\dot\alpha}^A\mathcal{W} = 0\,, \qquad D^{\alpha\,A}D_\alpha^B\mathcal{W} + \bar D_{\dot\alpha}^A\bar D^{\dot\alpha\,B}\bar{\mathcal{W}} = 0\,. \tag{4.6}
$$

Performing the appropriate translation $y^m = x^m - \mathrm{i}\theta_A\sigma^m\bar\theta^A$, and imposing the aforementioned chirality and reality conditions, we obtain an expansion of $\mathcal{W}$ in terms of $\mathcal{N}=1$ superfields

$$
\mathcal{W} = \phi(y,\tilde\theta) + \sqrt{2}\theta^\alpha W_\alpha(y,\tilde\theta) + \theta\theta G(y,\tilde\theta)\,, \tag{4.7}
$$

where

$$
G(y,\tilde\theta) = \int d^2\bar{\tilde\theta}\,\bar\phi(y + \mathrm{i}\tilde\theta\sigma\bar{\tilde\theta}, \bar{\tilde\theta})e^{2V(y+\mathrm{i}\tilde\theta\sigma\bar{\tilde\theta},\bar{\tilde\theta})}\,, \tag{4.8}
$$

and V is the prepotential of $W_\alpha = \mathrm{i}\bar{\tilde D}^2\tilde D_\alpha V$. These superfields transform in the usual way under the $\tilde Q, \bar{\tilde Q}$ generators but transform into each other under $Q, \bar Q$ as components of a chiral multiplet, in other words according to

$$
\begin{aligned}
\delta\phi &= \sqrt{2}(\xi W)\,, \\
\delta W_\alpha &= -\mathrm{i}\sqrt{2}(\sigma^m\bar\xi)_\alpha\partial_m\phi + \sqrt{2}\xi_\alpha G\,, \\
\delta G &= \mathrm{i}\sqrt{2}\partial_m W\sigma^m\bar\xi\,.
\end{aligned}
\tag{4.9}
$$

The usual free $\mathcal{N}=2$ super-Maxwell action constructed with the $\mathcal{N}=2$ superfield is

$$
S_2 = \int d^4x\,d\Theta\,\mathcal{W}^2 + \int d^4x\,d\bar\Theta\,\bar{\mathcal{W}}^2\,. \tag{4.10}
$$

Integrating over θ we obtain the following lagrangian density

$$
\frac{1}{4}\int d^2\tilde\theta\,W^2 + \frac{1}{4}\int d^2\bar{\tilde\theta}\,\bar W^2 - \int d^4\tilde\theta\,\bar\phi e^{2V}\phi\,, \tag{4.11}
$$

which is manifestly $\mathcal{N}=1$, gauge ($\delta V = \mathrm{i}(\Lambda - \bar{\Lambda})$) invariant and $\mathcal{N}=2$ via the transformation law (4.9). To relate the Goldstone supermultiplet with the $\mathcal{N}=2$ superfield we proceed in analogy to the previous case, identifying

$$\psi_\alpha = \lambda \mathcal{D}_\alpha \Phi\,, \tag{4.12}$$

and building a set of covariant constraints that reduce to (4.6) in the flat limit

$$\tilde{\bar{\mathcal{D}}}_{\dot\alpha}\Phi = 0\,, \tag{4.13a}$$

$$\bar{\mathcal{D}}_{\dot\alpha}\Phi + \frac{1}{4}\lambda^2 \bar{\mathcal{D}}_{\dot\alpha}\bar{\Phi}\bar{\mathcal{D}}^2(D\Phi)^2 = 0\,. \tag{4.13b}$$

This covariant chirality conditions are solved by

$$\Phi = \mathcal{W} - \mathrm{i}\lambda^2(D\mathcal{W}\sigma^m\bar{D}\bar{\mathcal{W}})\partial_m\mathcal{W} + \lambda^2 X + \mathrm{O}(\lambda^4)\,. \tag{4.14}$$

Where X is some $\mathcal{N}=2$ chiral superfield $\bar{D}_{\dot\alpha}X = \tilde{\bar{D}}_{\dot\alpha}X = 0$. Considering that λ has dimensions of L^{-2}, X cannot be trivially $\mathcal{W}$, instead X is of third order in $\mathcal{W}$. As it has been pointed out in other cases [6], the existence of dimensionless invariants $\bar{\mathcal{D}}^A_{\dot\alpha}\mathcal{D}_\beta\Phi$ and $\mathcal{D}^A_\alpha\mathcal{D}_\beta\Phi$ has a direct impact on the uniqueness of the constraints. In principle we could add any power of the dimensionless invariants without spoiling the flat limit $\lambda \to 0$. Keeping this in mind, we see that the remaining reality conditions, are generalized to

$$\mathcal{D}^{\alpha\,A}\mathcal{D}^B_\alpha\Phi + \bar{\mathcal{D}}^A_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha\,B}\bar{\Phi} - \lambda^2(\mathcal{D}^{\alpha\,A}\mathcal{D}^B_\alpha X + \bar{\mathcal{D}}^A_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha\,B}\bar{X}) + \lambda^2 f^{AB}(\Phi,\bar{\Phi}) = 0\,, \tag{4.15}$$

where $f^{AB}(\Phi,\bar{\Phi})$ is a function determined by an iterative procedure such that (4.14) solves indeed the former conditions. We notice that X is so far only restricted by the chirality conditions but not by the reality ones. The restrictions (4.15) reduce in the $\lambda \to 0$ limit to the usual reality conditions of the $\mathcal{N}=2$ field $\mathcal{W}$ defined by (4.6). The non-linear transformation law that realize a third supersymmetry on Φ can be derived from (2.4) and (4.12)

$$\delta\Phi = \theta\xi - \mathrm{i}\lambda^2(\xi\sigma^m\bar{D}\bar{\Phi} - D\Phi\sigma^m\bar{\xi})\partial_m\Phi + \mathrm{O}(\lambda^4)\,. \tag{4.16}$$

This transformation law fulfills the constraints (4.15) above and, as in the former case, closes in the supersymmetry algebra

$$[\delta_\xi, \delta_\eta]\Phi = 2\mathrm{i}\lambda^2(\xi\sigma^m\bar{\eta} - \eta\sigma^m\bar{\xi})\partial_m\Phi + \mathrm{O}(\lambda^4)\,. \tag{4.17}$$

Choosing $X=0$, an invariant action for Φ is

$$S = \int d^4x\, d\Theta\, \left[\mathrm{Ber}\,\Phi^2 - 2\mathrm{i}\lambda^2(\partial_m D\Phi\sigma^m\bar{D}\bar{\Phi})\,\Phi^2\right] + \frac{a\lambda^2}{2}\int d^4x\, d\Theta\, d\bar{\Theta}\Phi^2\bar{\Phi}^2 + \mathrm{c.c.} \tag{4.18}$$

Here the berezinian is included up to order λ^2

$$\mathrm{Ber} = 1 + \mathrm{i}\partial_m\psi\sigma^m\bar{\psi} - \mathrm{i}\psi\sigma^m\partial_m\bar{\psi}\,. \tag{4.19}$$

Where a is a constant factor. The reason to introduce the additional term in the action beyond the one involving the berezinian is the presence of the term $\theta\xi$ in the transformation law of Φ which is not a superfield under the linear supersymmetry transformations. For $a =$

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$-\frac{1}{4}$, the action (4.18) is invariant under the linearized $\mathcal{N}=1$ and $\mathcal{N}=2$ supersymmetries and under the transformation law (4.16), which in terms of the $\mathcal{N}=2$ chiral superfield reads

$$\hat{S}_2 = \int d^4x\, d\Theta\, \mathcal{W}^2 + \int d^4x\, d\bar{\Theta}\, \bar{\mathcal{W}}^2 - \frac{1}{4}\lambda^2 \int d^4x\, d\Theta\, d\bar{\Theta}\, \mathcal{W}^2\bar{\mathcal{W}}^2 + \mathrm{O}(\lambda^4)\,. \qquad (4.20)$$

The action (4.20) is self dual in our approximation. Moreover, (4.20) is the $\mathcal{N}=2$ supersymmetric low energy Born-Infeld action, that coincides up to fourth order in the superfields with that proposed by Ketov [23, 24], as the $\mathcal{N}=2$ supersymmetric extension of the 4-dimensional Born-Infeld action, that found by Bellucci, Ivanov and Krivonos [3] by PSSB of $\mathcal{N}=4 \longrightarrow \mathcal{N}=2$ with central charges, and that found by Kuzenko and Theisen [10] up to order $\mathrm{O}(F^8)$ by requesting self-duality. Hence this action fulfills the requirements for describing the dynamics of a single D3-brane in six dimensions. The whole analysis may be now performed for $X \neq 0$. A general choice of the chiral field X in terms of $\mathcal{W}$ has the form

$$X = b\bar{D}^2\bar{\tilde{D}}^2\bar{\mathcal{W}}^3\,, \qquad (4.21)$$

where b is a dimensionless constant. Including $X \neq 0$ we obtain the following action

$$\hat{S}_2 = \int d^4x\, d\Theta\, \mathcal{W}^2 + \lambda^2 \int d^4x\, d\Theta\, d\bar{\Theta}\, [a\mathcal{W}^2\bar{\mathcal{W}}^2 + 2b\mathcal{W}\bar{\mathcal{W}}^3] + \text{c.c.} + \mathrm{O}(\lambda^4)\,, \qquad (4.22)$$

which is invariant under the non-linear transformation law (4.16). The parameter b can not be absorbed into $\mathcal{W}$ by the redefinition

$$\widetilde{\mathcal{W}} = \mathcal{W} + \lambda^2 X$$

since, though X is chiral, it is not restricted by any reality condition so that (4.22) comprises a plethora of dynamics. This is not completely unexpected since the curve reality constraints on Φ are X dependent.

5. Concluding remarks

In the first part of this work we found a family of non-linear transformation laws realizing an extra supersymmetry on the chiral $\mathcal{N}=1$ superfield, up to second order in the scale of the supersymmetry breaking, obtained from the non-linear realizations method. We imposed some restrictions over the most general set of variations, asking for the subset that could be considered as second supersymmetries non-linearly realized, and choosing the ones that fulfill the $\mathcal{N}=2$ algebra. Moreover, we were able to find the action constructed in [12] as a particular case. In another case, we find the action in [20] up to $\mathrm{O}(\varphi^4)$ which is the translational invariant action of the $\mathcal{N}=1$ 3-brane proposed in [21]. Instead of making cumbersome field redefinitions, we move through the set of lagrangians by selecting the values of the parameters a_i. In the second part, we constructed a family of $\mathcal{N}=2$ actions invariant under a third broken hidden supersymmetry, considering the chiral $\mathcal{N}=2$ superfield as the Goldstone field coming from the partial supersymmetry breaking of a $\mathcal{N}=3$ theory. This actions are self dual up to $\mathrm{O}(\lambda^2)$. In a specific case, the lagrangian is the $\mathcal{N}=2$ supersymmetric Born-Infeld up to $\mathrm{O}(F^4)$ that describes the world volume

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dynamics of a single D3-brane propagating in six dimensions. It is important to stand out that this result and those in the literature [2]–[6] and [9]–[11], provide examples of a supersymmetry Born-Infeld theory that arises from the partial supersymmetry breaking. It seems to be that the nature of this SUSY non-linear electromagnetic dynamics comes from the partially breaking of higher supersymmetries.

Though the non-linear realization formalism was carried out in both cases completely without central charges, and the geometrical objects involved are independent of them, the resulting algebras reveal the presence of a hidden central charge. Owing to the generality of the off-shell procedure followed, we presume that for the cases here studied, it is not possible to break the supersymmetry without central charges. These should be associated to the maximal automorphism group [12].

Due to the preservation of 2/3 of the supersymmetry, some non-BPS D-brane dynamics could arise from the breaking of $\mathcal{N}=3 \longrightarrow 2$, as suggested by E. Ivanov [26]. This situation could already be present in our family of lagrangians or in another Goldstone multiplet selection.

In the context of string theory it is also important to find the correct non-abelian supersymmetric Born-Infeld functional. Some progress in this direction has been made in [27, 28]. It is still unknown if there exists a Goldstone multiplet coming from PSSB that produces a non-abelian supersymmetric Born-Infeld theory.

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A. Notation and conventions

We use the following signature for the metric and antisymmetric tensors

$$\eta_{mn} = \mathrm{diag}(-+++)\,, \qquad \varepsilon_{\alpha\beta} = \varepsilon^{\dot\alpha\dot\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{A.1}$$

The Pauli matrices and superspace derivatives are taken in the following representation

$$\sigma^m = \left[-\mathbb{1},\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix},\ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]. \tag{A.2}$$

$$D_\alpha = \partial_\alpha - \mathrm{i}(\sigma^m\bar\theta)_\alpha\partial_m, \quad \bar D_{\dot\alpha} = -\bar\partial_{\dot\alpha} + \mathrm{i}(\theta\sigma^m)_{\dot\alpha}\partial_m\,. \tag{A.3}$$

The anticommutator between derivatives will then be

$$\{D_\alpha, \bar D_{\dot\beta}\} = 2\mathrm{i}\sigma^m_{\alpha\dot\beta}\partial_m\,. \tag{A.4}$$

In general, we follow the conventions in [29] for the contraction of the spinorial indexes.

B. A family of $N=2$ non-linear transformation laws

In principle, the most general superfield Z_ξ containing φ that is dimensionally consistent is

$$\begin{aligned}Z_\xi = {} & b_1\varphi\xi^\alpha D_\alpha\varphi + b_2\varphi\bar\xi_{\dot\alpha}\bar D^{\dot\alpha}\bar\varphi + b_3(\theta\xi)(D\varphi)^2 + b_4(\bar\theta\bar\xi)(D\varphi)^2 + b_5\bar\xi_{\dot\alpha}\bar D^{\dot\alpha}\bar\varphi\theta^\alpha D_\alpha\varphi + \\ & + b_6\xi^\alpha D_\alpha\varphi\bar\theta_{\dot\alpha}\bar D^{\dot\alpha}\bar\varphi + b_7(\theta\xi)(\bar D\bar\varphi)^2 + b_8(\bar\theta\bar\xi)(\bar D\bar\varphi)^2 + b_9(\theta\xi)D^2\varphi\varphi + b_{10}(\theta\xi)D^2\varphi\bar\varphi + \\ & + b_{11}(\bar\theta\bar\xi)D^2\varphi\varphi + b_{12}(\theta\xi)\bar D^2\bar\varphi\varphi + b_{13}(\theta\xi)\bar D^2\bar\varphi\bar\varphi + b_{14}(\bar\theta\bar\xi)\bar D^2\bar\varphi\varphi + \\ & + b_{15}(\bar\theta\bar\xi)\bar D^2\bar\varphi\bar\varphi + b_{16}(\theta\sigma^m\bar\xi)\partial_m\varphi\varphi + b_{17}(\theta\sigma^m\bar\xi)\partial_m\varphi\bar\varphi + b_{18}(\xi\sigma^m\bar\theta)\partial_m\varphi\varphi + \\ & + b_{19}(\xi\sigma^m\bar\theta)\partial_m\varphi\bar\varphi + b_{20}(\theta\sigma^m\bar\xi)\partial_m\bar\varphi\varphi + b_{21}(\xi\sigma^m\bar\theta)\partial_m\bar\varphi\varphi\,, \end{aligned} \tag{B.1}$$

where b_j are constants to be determined. It is easy to see that $\mathcal{X}$ of equation (3.7) satisfies $\bar D^2[\delta_\xi,\delta_\eta]\mathcal{X}=0$ so according to (3.6a) Z_ξ must satisfy

$$D^2\delta_{[\xi}Z_{\xi]} = 0\,, \tag{B.2}$$

this restricts the field to

$$\begin{aligned} b_2+b_5+b_{20} &= 2b_7\,, \\ b_{13} &= b_{14}\,, \\ b_{16}=b_{17}=b_{18}=b_{19} &= 0\,. \end{aligned} \tag{B.3}$$

The other constraint (3.6b) cancels the remaining terms in Z_ξ. As $\mathcal{X}$ is not restricted, we have found in (3.5b) a family of non-linear transformations realizing an extra supersymmetry over the chiral $N=1$ action.

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